

Experimental investigation on a thermoacoustic engine having a looped tube and resonator

Z.B. Yu ^a, Q. Li ^{a,*}, X. Chen ^b, F.Z. Guo ^b, X.J. Xie ^a

^a Cryogenics Laboratory, Technical Institute of Physics and Chemistry, Chinese Academy of Sciences, P.O. Box 2711, Beijing 100080, China

^b Cryogenics Laboratory, Huazhong University of Science and Technology, Wuhan 430074, China

Received 18 November 2003; received in revised form 22 March 2005; accepted 7 June 2005

Abstract

The purpose of this paper is to study the impact of regenerator hydraulic radius, resonator length, and mean pressure on the characteristics of the tested thermoacoustic engine, which has a looped tube and resonator. Two different acoustic oscillations are observed in the tested engine [Yu ZB, Li Q, Chen X, Guo FZ, Xie XJ, Wu JH. Investigation on the oscillation modes in a thermoacoustic stirling prime mover: mode stability and mode transition. *Cryogenics* 2003;43(12):687–91]. In this paper, they are called two acoustic modes, high frequency mode (with a frequency independent of the resonator length) and low frequency mode (with a frequency depending on the resonator length). Experimental results indicate that the relative penetration depth (the ratio of penetration depth over hydraulic radius) plays an important role in the excitation and pressure amplitude of the two acoustic modes. For each tested regenerator hydraulic radius, there is a measured optimal relative penetration depth, which leads to the lowest onset temperature difference. Note that, in the tested engine, the measured optimal relative viscous penetration depths are in the range 3–5 (for low frequency mode). Furthermore, experimental results also show that the resonator length affects the presence of high frequency mode in this engine.

© 2005 Elsevier Ltd. All rights reserved.

Keywords: Thermoacoustic engine; Relative penetration depth; Acoustic mode

1. Introduction

A thermoacoustic engine (TAE) consisting of a looped tube and resonator, with the thermoacoustic core in the looped tube, was for the first time constructed and studied by Backhaus and Swift [2,3]. It was called thermoacoustic Stirling engine. With Helium as working gas, the engine worked at a frequency of 80 Hz, which was a quarter-wavelength mode. At its most efficient operating point, the efficiency was 0.30. In 2001, Biwa et al. also described a TAE of this type [4–6]. Two

modes, traveling wave mode (273 Hz) and standing wave mode (100 Hz), were observed in their system. Furthermore, with increasing heat supplied to the engine they found a transition from standing to traveling wave mode through the quasi-periodic states involving both standing and traveling wave modes [4,5]. In 2002, Ueda et al. [7,8] constructed another TAE of this kind with Pyrex glass tubes. They measured pressure and velocity simultaneously. The experimental results indicated that the phase lead φ of U relative to p in the regenerator was -20° , but not a traveling wave phase ($\varphi = 0^\circ$) as reported by Backhaus and Swift [2,3]. Moreover, they studied experimentally the optimization of the length ratio of the resonator and the looped tube as well as the position of the connecting point between the resonator and the looped tube.

* Corresponding author. Tel.: +86 10 626 10 915; fax: +86 10 625 64 049.

E-mail address: liqing@cl.cryo.ac.cn (Q. Li).

We also constructed a TAE having a looped tube and resonator in 2001 [9]. Two different acoustic modes, high frequency mode (HFM, 528 Hz) and low frequency mode (LFM, 76 Hz), were observed in this TAE, when the working gas is nitrogen [1]. The transition from HFM to LFM was investigated in detail for a 150-mesh regenerator. The results indicated that the mean pressure p_m was an important control parameter influencing the mode stability in the TAE. (Note that the “modes” mentioned in this paper are defined only by the different frequencies which distinguish the two kinds of oscillations in the tested TAE. HFM denotes the oscillation with a frequency independent of the resonator length, while LFM denotes the oscillation with a frequency depending on the resonator length.)

In this paper, we concentrate on the effect of the regenerator hydraulic radius r_h , resonator length, and mean pressure on the excitation and performance of the two modes. Note that the role of the relative penetration depth was studied in detail. Furthermore, the presence of the HFM and the means to eliminate it in the tested engine are studied and discussed.

2. Experimental apparatus

The tested TAE is schematically shown in Fig. 1. Its details can be found in previous papers [1,9,10]. The resonator includes a straight stainless-steel pipe and a cavity, which are of 35 and 70 mm inner diameter, respectively. The length of the straight pipe L_2 can be

changed. The mean length of the looped tube (L_1) is 0.708 m. The regenerator is a pile of stainless steel gauze, the mesh of which is given by the manufacturer as shown in Table 1. Five piezoelectric pressure sensors for measuring the pressure amplitudes are placed in the looped tube and indicated by P_1 – P_5 in Fig. 1. Four thermocouples (T_1 – T_4) are equidistantly placed along the length of the regenerator from the hot to the cold end [1].

3. Experimental results

3.1. Regenerator hydraulic radius

In the following experiments, the working gas was nitrogen. Its mean pressure p_m was varied from 0.1 to 2.6 MPa (absolute pressure), with steps of 0.1 MPa. The resonator length was fixed at 1.138 m. We measured the relationship between onset temperature difference ΔT_{onset} and p_m by varying the hydraulic radius r_h . It was found that the minimum heating power needed to make the tested TAE oscillate varied when either the mean pressure, the working gas, or r_h were changed. We chose a heating power (208 W), which makes the tested TAE oscillate at all test conditions, for obtaining comparable experimental results over the full variation range of these operating conditions.

For a 60-mesh regenerator, two measured curves are shown in Fig. 2(a). The experimental phenomena were similar to those of the 150-mesh regenerator investigated previously [1]. There was a critical mean pressure $p_{m,\text{cri}}$ (about 0.24 MPa), above which the HFM (536 Hz) could be observed, and below which the engine began to oscillate in the LFM (76 Hz), and then reached stationary conditions only if the temperature difference was strictly controlled in the region between the two intersecting curves. Otherwise, the oscillating mode would change from HFM to LFM through the quasi-periodic state involving both HFM and LFM. On the other hand, there was an optimal mean pressure $p_{m,\text{opt}}$,

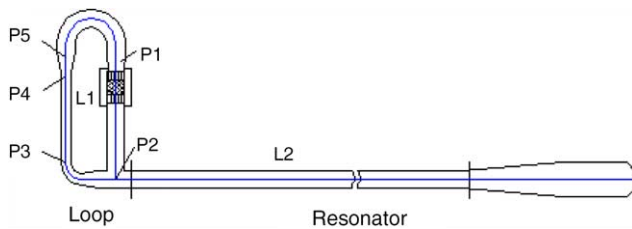


Fig. 1. Schematic illustration of the tested TAE.

Table 1

Measured critical mean pressure, optimal mean pressure, lowest onset temperature difference, and optimal relative viscous penetration depth $(\delta_v/r_h)_{\text{opt}}$ when the r_h of the regenerator is varied

Mesh	60 ^a	80	120	150	200	250	300	350
Hydraulic radius r_h (μm)	97.3	71.1	47.4	37.7	27.2	22.3	18.2	15.6
Critical mean pressure ^b $p_{m,\text{cri}}$ (MPa)	0.24	0.45	0.6	1.0	1.3	2.3	–	–
Optimal mean pressure $p_{m,\text{opt}}$ (MPa)	– ^c	0.26	0.4	0.6	0.6	0.8	1.6	2.1
ΔT_{low} (K)	–	209	174	226	149	139	105	314
$(\delta_v/r_h)_{\text{opt}}$	–	3.0	3.6	3.8	4.6	4.7	4.1	5.4

^a Meaning 60 wires per inch in each of the two perpendicular directions of the weave.

^b Absolute pressure.

^c Not observed.

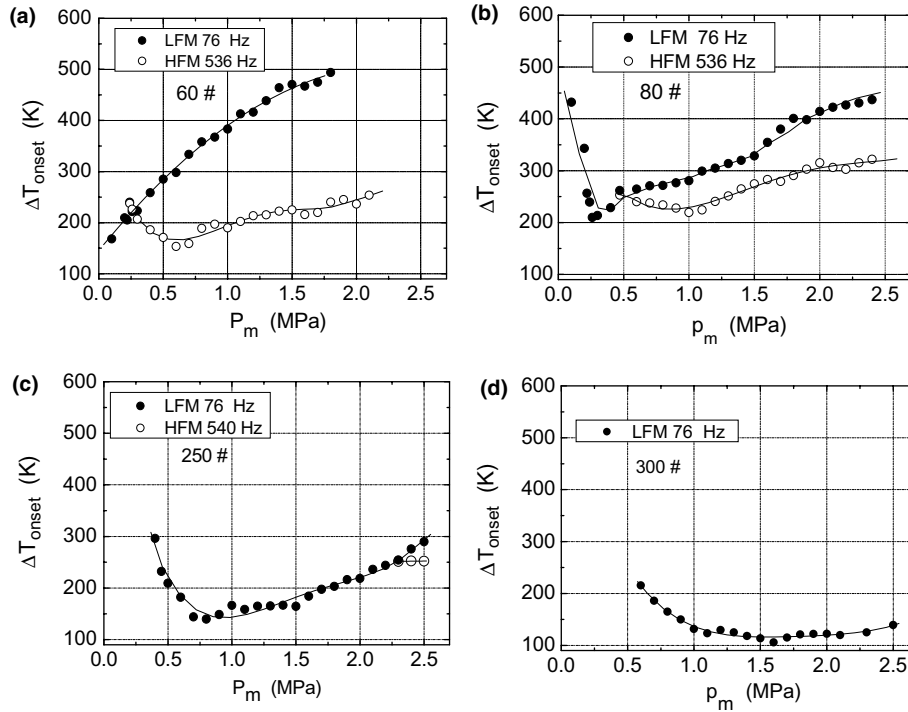


Fig. 2. Measured relationship between ΔT_{onset} and p_m of the tested TAE with varying regenerator hydraulic radius. The lines are guides for the eye.

which led to the lowest onset temperature difference ΔT_{low} for LFM.

By the same means, we obtained the curves of other regenerators. The experimental results are shown in Fig. 2(b) (80-mesh), Fig. 2(c) (250-mesh), Fig. 2(d) (300-mesh). The measured $p_{m,\text{cri}}$, $p_{m,\text{opt}}$ and ΔT_{low} versus r_h are shown in Table 1. Note that for the 300-mesh and 350-mesh regenerator HFM was not observed throughout the tested pressure range. Fig. 2 indicates that HFM only appears at mean pressures above $p_{m,\text{cri}}$. From Table 1, one can find that $p_{m,\text{cri}}$ and $p_{m,\text{opt}}$ increase with r_h decreasing.

In regenerators, viscous and thermal penetration depths are defined by $\delta_v = \sqrt{2\mu/\omega\rho_m}$ and $\delta_k = \sqrt{2K/\omega\rho_m c_p}$, where μ is the gas dynamic viscosity, c_p is the gas isobaric heat capacity per unit mass, K is the gas heat conductivity, ρ_m is the mean gas density, and ω is the angle frequency of the gas oscillation. In this paper, the relative viscous (or thermal) penetration depth is defined by δ_v/r_h (or δ_k/r_h), where r_h is the hydraulic radius of the regenerator. Obviously, for a given frequency, both the decrease of p_m and r_h cause an increase of the relative penetration depth. We choose the relative viscous penetration depth δ_v/r_h of the LFM for further discussion in the following text.

Each onset point shown in Fig. 2 corresponds to a certain value of δ_v/r_h . We use ΔE to obtain δ_v of each point. Three parameters, measured temperature, measured frequency, and mean pressure, are needed. The mean temperature along the length of the regenerator $\bar{T} = (T_1 + T_2 + T_3 + T_4)/4$ is taken as measured temperature.

Thus, the δ_v/r_h is a mean value along the length of the regenerator. For example, the measured results in Fig. 2(a) were transformed into the plot of ΔT_{onset} versus δ_v/r_h taking the form of two stability curves shown in Fig. 3(a). By the same means, Fig. 3(b)–(d) are obtained.

From Fig. 3, it is found that there is an optimal value of δ_v/r_h which leads to the lowest onset temperature difference for the two acoustic modes, respectively (see Fig. 3(a) and (b)). The optimal δ_v/r_h increases slightly with the hydraulic radius decreasing. Furthermore, the stability curves of each mode are approximately symmetrically distributed at both sides of the optimal δ_v/r_h value (see Fig. 3(b) and (c)).

Generally, in a regenerator, the thermal (or viscous) penetration depth should be much larger than the hydraulic radius to maintain a good thermal contact between the working gas and the solid heat capacity [11]. From Fig. 3 and Table 1, in this engine, the measured optimal value δ_v/r_h was in the range 3–5 for the LFM. The lowest onset temperature difference obtained in these experiments was 105 K (see Fig. 3(d)).

Furthermore, from Fig. 3, one can find that the appropriate relative penetration depths required for HFM decrease as r_h decreases (see Fig. 3). This reveals the reason why the decrease of both p_m and r_h can help to eliminate HFM in the tested engine.

3.2. Working gas

The experimental results presented in Section 3.1 indicate that the relative penetration depth has a

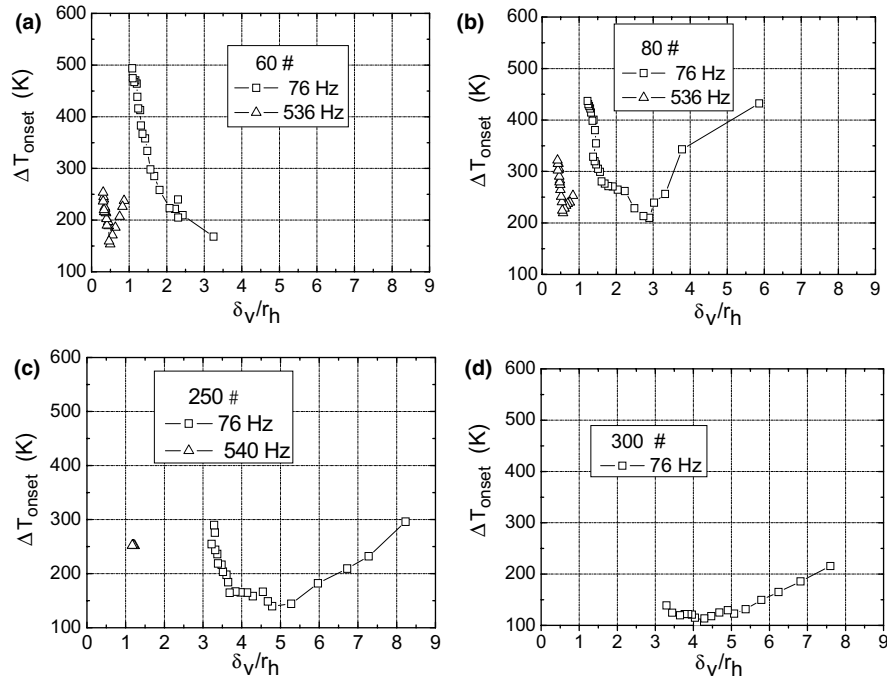


Fig. 3. The relationship between ΔT_{onset} and δ_v/r_h with varying regenerator hydraulic radius. The lines are guides for the eye.

significant effect on the characteristics of the tested system. Especially, there is a measured optimal δ_v/r_h , which leads to the lowest onset temperature difference. The experiments in this section were performed to investigate the optimal value of δ_v/r_h when the working gas was changed. Helium was studied for comparison with nitrogen.

Using the same experimental method and conditions as described in Section 3.1, the 120-mesh and 150-mesh regenerators were studied experimentally with helium and nitrogen, respectively, as working gas. The measured results are shown in Fig. 4(a), while Fig. 4(b) shows the transformed curves. Note that the HFM is not considered in Fig. 4. By the way, when the working gas is helium, the two frequencies of the tested TAE are 211 Hz and 1406 Hz respectively. It can be seen from

Fig. 4(a) that the optimal mean pressures are quite different from each other. However, Fig. 4(b) shows that the optimal values of δ_v/r_h are all close to 4.

3.3. Amplitude properties

In the experiments described in Sections 3.1 and 3.2 (nitrogen as working gas), we also measured the magnitude of the pressure amplitudes at five points (P_1 – P_5). It was found that the pressure amplitude at P_1 (just above the cold heat exchanger) was the biggest one among the five when the system oscillated in LFM. We choose the pressure amplitude of P_1 (p_1) for analysis in this section. Fig. 5 shows the relationship between the measured pressure amplitudes and the mean pressure for each regenerator.

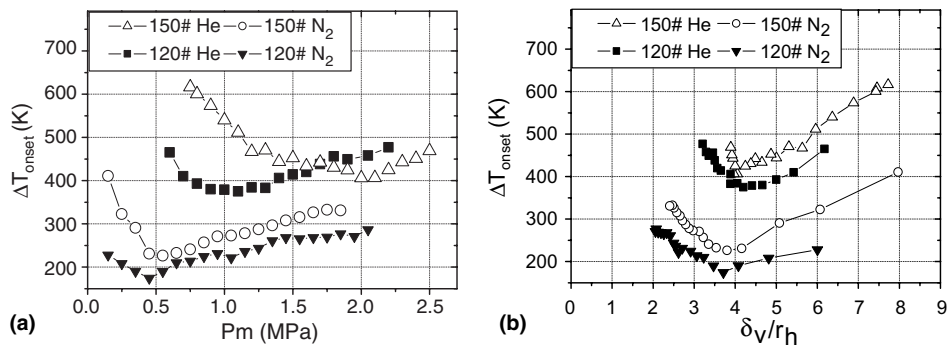


Fig. 4. (a) The measured relationship between ΔT_{onset} and p_m for the working gases nitrogen and helium, respectively. (b) The relationship between ΔT_{onset} and δ_v/r_h .

Firstly, Fig. 5 indicates that the amplitude of p_1 increases as p_m increases, although with decreasing speed. Secondly, when $p_m \leq 0.6$ MPa the amplitude of p_1 can be enhanced by increasing r_h , while the amplitude of p_1 decreases with an increase of r_h when $p_m \geq 1.5$ MPa. Furthermore, it is interesting that the measured results of the 300-mesh regenerator are between those of the 200-mesh and 250-mesh regenerators. However, the lowest measured value of ΔT_{onset} (105 K) was obtained when the regenerator is 300-mesh. The further mechanism is not clear.

3.4. Resonator lengths

The following experiments were performed to investigate the impact of the resonator length on the system. For all measurements an 80-mesh regenerator was used. Five different resonator lengths were obtained by connecting three pipes of different length in different combinations. For each resonator length the critical mean pressure and the frequencies of the HFM and LFM were measured. The results are shown in Table 2.

It can be seen that the frequency of HFM was independent of the resonator length, while the frequency of LFM depended on the resonator length. Furthermore, no HFM was observed throughout the tested mean pressure range when L_2 is 1.808 m. Obviously, the resonator length affects the excitation of the HFM significantly.

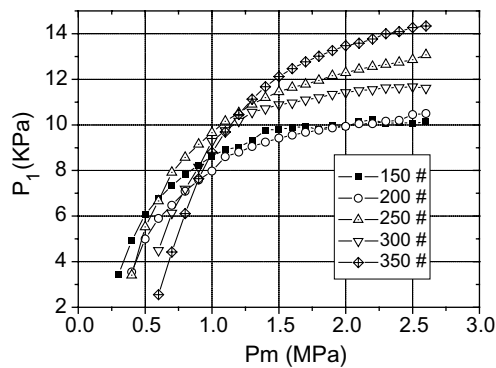


Fig. 5. Measured amplitude of p_1 with variation of mean pressure and regenerator hydraulic radius.

Table 2
Measured critical mean pressure and HFM and LFM frequencies of the tested TAE when the resonator length is varied

L_2 (m)	0.678	1.138	1.808	2.238	2.668
$p_{m, \text{cri}}^a$ (MPa)	— ^b	0.45	—	2.4	1.9
f_{HFM} (Hz)	537	536	—	536	534
f_{LFM} (Hz)	—	76	57	49	44

^a Absolute pressure.

^b Not observed.

4. Discussion

Basing on the experimental results and their analysis, the main facts resulting from our investigation are the following:

- (1) In the tested pressure range, the HFM is only excited at relatively high mean pressures above $p_{m, \text{cri}}$. Furthermore, $p_{m, \text{cri}}$ increases as the hydraulic radius of the regenerator decreases (see Table 1). In other words, decreasing p_m and r_h helps to eliminate the HFM in the tested system. Because decreasing of p_m and r_h at a given oscillating frequency both cause an increase of the relative penetration depth, it can be concluded that the range of δ_v/r_h required for HFM to be excited obviously decreases with p_m and r_h decreasing, and thus the HFM is suppressed.
- (2) For each regenerator, an optimal mean pressure value $p_{m, \text{opt}}$ was measured, which leads to the lowest onset temperature difference in the tested TAE. Furthermore, $p_{m, \text{opt}}$ increases as r_h decreases (see Fig. 2 and Table 1).
- (3) Although the optimal mean pressures $p_{m, \text{opt}}$ are quite different from each other (see in Fig. 2 and Table 1), the corresponding optimal values of δ_v/r_h are very close to each other and in the range 3–5 (see Fig. 3 and Table 1), and this holds even for different working gases. For δ_k/r_h the measured optimal value is in the range 4–7. These experimental results reveal that the relative penetration depth plays a key role in the excitation of HFM or LFM in the tested engine. By the way, in the most efficient TAE described by Backhaus and Swift, with a regenerator hydraulic radius $r_h \approx 42 \mu\text{m}$, the measured thermal penetration depth of helium δ_k is 140–460 μm throughout the regenerator [3]. This operating condition approximately corresponds to our measured optimal range. The experimental results presented here indicate that the regenerator can be optimized in a practical TAE by choosing an appropriate value of relative penetration depth, when the working gas, mean pressure, and operating frequency are given.
- (4) The lowest onset temperature difference found in our experiments is 105 K with a regenerator of 300-mesh (see Fig. 2(d)). Such a low onset temperature difference demonstrates the advantage of TAE in the utilization of low-level heat energy. The largest pressure amplitude of 14.5 kPa was observed with a regenerator of 350-mesh (see Fig. 4).
- (5) In the tested TAE, the resonator length affects the occurrence of the HFM significantly. Note that the experimental result of Ueda et al. [8] also indicated that the length ratio of the looped tube and the

resonator plays an important role in the excitation of acoustic modes in the TAE having a looped tube and a resonator, although the mechanism is still unclear.

5. Conclusion

A series of experiments have been performed to investigate the impact of the regenerator hydraulic radius, resonator length, mean pressure, and working gas on the characteristics of the tested TAE. The experimental results reveal that the relative penetration depth plays an important role in the excitation of the system. There is a measured optimal value of δ_v/r_h in the range 3–5 for the LFM. On the other hand, the experimental results also indicate that more attention should be paid to the length ratio of the looped tube over the resonator in the design of TAEs of this kind.

In the future, we plan to further optimize the design by using different mesh stainless steel screen in a regenerator to match the temperature gradient along it. Furthermore, the mechanism of the coupling between the resonator and the looped tube will be investigated experimentally and theoretically.

Acknowledgements

This work was supported by the National Natural Science Foundation of China under Grant No. 90407012 and Knowledge Innovation Program of the CAS under Grant No. KJCX2-SW-W08.

All the authors express thanks to Mr. Tetsushi Biwa for the discussion of the experimental results.

References

- [1] Yu ZB, Li Q, Chen X, Guo FZ, Xie XJ, Wu JH. Investigation on the oscillation modes in a thermoacoustic stirling prime mover: mode stability and mode transition. *Cryogenics* 2003;43(12): 687–91.
- [2] Backhaus S, Swift GW. A thermoacoustic Stirling heat engine. *Nature* 1999;399:335–8.
- [3] Backhaus S, Swift GW. A thermoacoustic stirling heat engine: detailed study. *J Acoust Soc Am* 2000;107(6):3148–66.
- [4] Biwa T, Ueda Y, Yazaki T, Mizutani U. Thermodynamical mode selection rule observed in spontaneous gas oscillations. In: The proceedings of 17th international conference on acoustics, Part A. Rome: 2001.
- [5] Biwa T, Ueda Y, Yazaki T, Mizutani U. Thermodynamical mode selection rule observed in thermoacoustic oscillations. *Europhys Lett* 2002;60(3):363–8.
- [6] Biwa T, Ueda Y, Yazaki T, Mizutani U. Work flow measurements in a thermoacoustic engine. *Cryogenics* 2001;41:305–10.
- [7] Ueda Y, Biwa T, Mizutani U, Yazaki T. Acoustic field in a thermoacoustic Stirling engine having a looped tube and resonator. *Appl Phys Lett* 2002;81(27):5252–4.
- [8] Ueda Y, Biwa T, Mizutani U, Yazaki T. Experimental studies of thermoacoustic Stirling prime mover and its application to a cooler. *J Acoust Soc Am* 2004;115(3):1134–41.
- [9] Wu JH, Li Q, Zhang XQ, Yu ZB, et al. Investigation on the onset modes of oscillation in self-excited loop thermoacoustic engine. In: Proceedings of the 19th international cryogenic engineering conference. France: 2002. p. 347–50.
- [10] Li Q, Wu J, Guo FZ, Tu Q, Yu ZB. Investigation on a high frequency traveling wave thermoacoustic-driven system. In: Proceedings of the 12th international cryocoolers conference. USA: 2002. p. 439–46.
- [11] Swift GW. Thermoacoustics: a unifying perspective for some engines and refrigerators. *Acoustical Society of America*; 2002.